What is adaptive control?

- Now, although the purpose of the class is to learn this, we can contemplate an example that illuminates some of the questions and hopefully some of the challenges.

Start w/a dynamical system to control,

$$\dot{x} = f(\underset{\text{system state}}{x}, \underset{\text{controller}}{u})$$

where designed controller $u(t)$ leads to desired behavior in $x(t)$.

In reality, the system has some unknown or incorrectly estimated components/parameters

$$\dot{x} = f(x, u; \mu)$$

where $\mu \in P$ and the true values are $\mu^* \in P$ for $P \subset \mathbb{R}^p$ compact.

→ CAN CONTROL DESIGN USING μ STILL STABILIZE FOR $\mu \neq \mu^*$?

HOW LARGE CAN DOMAIN P BE?

HOW QUICKLY CAN WE COMPENSATE FOR $\mu \neq \mu^*$?

CAN WE STILL MEET PERFORMANCE SPECS?

Example: Uncertain Roll Dynamics

$$\dot{p} = \underbrace{L_p}\, \underbrace{p}_{\text{roll rate}} + \underbrace{L_\delta}_{\text{unknown damping \& aileron effectivenesses.}}\, \delta$$

aileron position (our control variable)

$\Rightarrow$

$$\dot{p} = L_\delta \left(\frac{L_p}{L_\delta} p + \delta \right)$$

$$\dot{p} = L_\delta (a p + \delta) \qquad \sim \qquad \dot{x} = a x + u$$

Scenario: suppose we choose state feedback to stabilize p.

$\Rightarrow$

$$\delta(t) = -k\, p(t)$$

$\Rightarrow$

$$\dot{p} = L_\delta (a-k) p = -L_\delta (k-a) p$$

Q: What choice of k will stabilize the system?

A: $k > a$

Q: What if true value $a^* \in P \equiv [a^*_{min}, a^*_{max}]$ is unknown?

A: $k > a^*_{max}$

Q: Now, what if at best, conservative estimates of a^*_{min} & a^*_{max} existed, e.g., $\hat{a}_{min}$ & $\hat{a}_{max}$ such that $[a^*_{min}, a^*_{max}] \subset [\hat{a}_{min}, \hat{a}_{max}]$?

A: $k > \hat{a}_{max}$

PROBLEMATIC !!! Why?

1] Whole procedure based on an estimate.

- What if $\hat{a}_{max}$ estimate ~~are~~ is not large enough and it was that $a^*_{max} > \hat{a}_{max}$?

System could be unstable.

2] Procedure is conservative.

- What if a^* ~~were~~ was equal to a^*_{min} 95% of the time?

Then too much control effort is being provided.

This is wasteful.

↳ mechanical wear

power issues (electric, fuel, etc.)

⋮

Adaptive control seeks to eliminate this problem while gauranteeing stability

this is tricky part!

but, robust control seeks to handle uncertainty or unmodeled components while gauranteeing stability. What's the difference?

anyhow, back to the problem of handling $a \neq a^*$ in an adaptive fashion. Here are two similar solutions:

$\boxed{1}$ select the following controller and adaptive law

$$\delta(t) = \delta_{ad}(t) \equiv k(t)\, p(t)$$

where

$$\dot{k}(t) = -\gamma\, p^2(t) \,, \qquad \gamma > 0$$

called: adaptation rate

We get the following cleverly written closed-loop system:

$$\dot{p} = L_\delta a_m p + L_\delta (k(t) + a - a_m) p \,, \qquad a_m < 0$$

- CAN SHOW STABILITY OF THIS SYSTEM.
- a_m IS CHOSEN TO MODEL DESIRED BEHAVIOR.
- SIGNALS BOUNDED IN THE PRESENCE OF UNCERTAINTIES.
- NOTICE THAT A NONLINEAR ADAPTIVE SYSTEM WAS OBTAINED FROM A LINEAR UNCERTAIN SYSTEM.

$\boxed{2}$ our second system begins with a desired model,

$$\dot{p}_m = L_{p,m}\, p_m + L_{\delta,m}\, \delta \qquad (*)$$

control input.

desired model parameters.

→ we want the roll-rate tracking error,

$$e_p(t) \equiv p(t) - p_m(t)$$

to vanish, e.g., $e_p(t) \to 0$ in time

So, goal is to design an adaptive control law $\delta_{ad}(t)$ such that

$$\dot{p} = L_p\, p + L_\delta\, \delta_{ad}$$

tracks (*) under the control input $\delta(t)$.

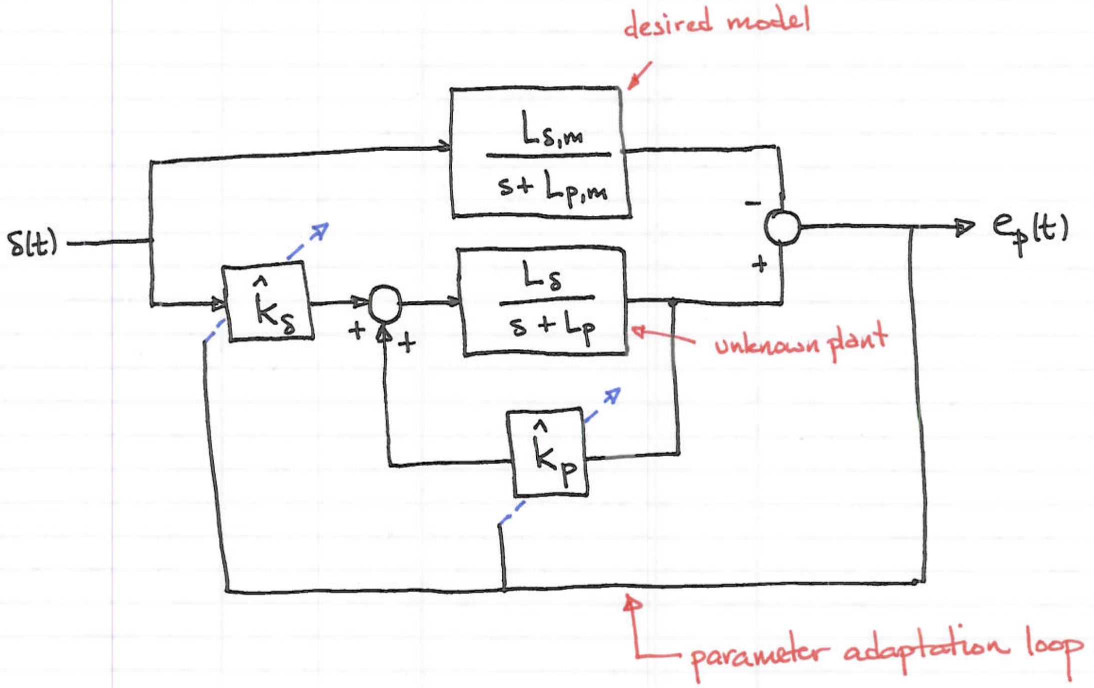
One such ~~adapt~~ adaptive law is:

$$\delta_{ad}(t) = \hat{k}_p(t)\, p(t) + \hat{k}_\delta\, \delta(t)$$

$$\dot{\hat{k}}_p(t) = -\gamma_p\, p(t)\,(p(t) - p_m(t)) \qquad \gamma_p > 0$$

$$\dot{\hat{k}}_\delta(t) = -\gamma_\delta\, \delta(t)\,(p(t) - p_m(t)) \qquad \gamma_\delta > 0$$

As a block diagram:



· YIELDS ASYMPTOTIC TRACKING WITH ALL REMAINING SIGNALS BOUNDED IN THE PRESENCE OF UNCERTAINTIES.