3/4/2010

Homogeneous Coordinates:

- A point p is given by the homogeneous coordinate q where

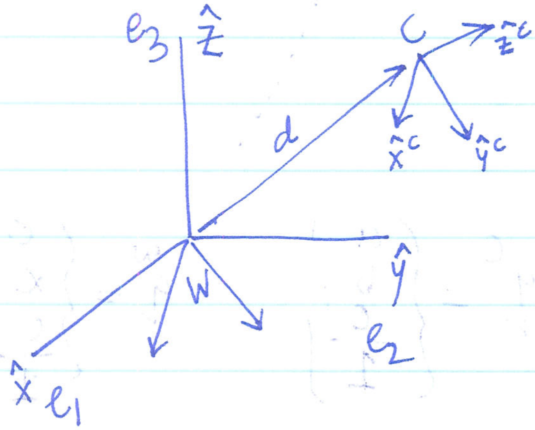
$$q = \left[\begin{array}{c} p \\ \hline 1 \end{array}\right]$$

- A configuration consisting of the rotation R and the translation given by the homogeneous matrix g, where

$$g = \left[\begin{array}{c|c} R & d \\ \hline 0 & 1 \end{array}\right]$$

$$p = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \longrightarrow q = \left[\begin{array}{c} x \\ y \\ z \\ \hline 1 \end{array}\right]$$

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \qquad d = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$



$$\left[\begin{array}{c|c} R & d \\ \hline 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{c|c} [3\times3] & [3\times1] \\ \hline [1\times3] & 1\times1 \end{array}\right] \rightarrow [4\times4]$$

$$\hat{x}^c = r_{11}\hat{x} + r_{21}\hat{y} + r_{31}\hat{z}$$

$$\hat{y}^c = r_{12}\hat{x} + r_{22}\hat{y} + r_{32}\hat{z}$$

$$\hat{z}^c = r_{13}\hat{x} + r_{23}\hat{y} + r_{33}\hat{z}$$

$$\left[\begin{array}{c|c} R & d \\ \hline 0 & 1 \end{array}\right] \rightarrow [4 \times 4] \text{ matrix}$$

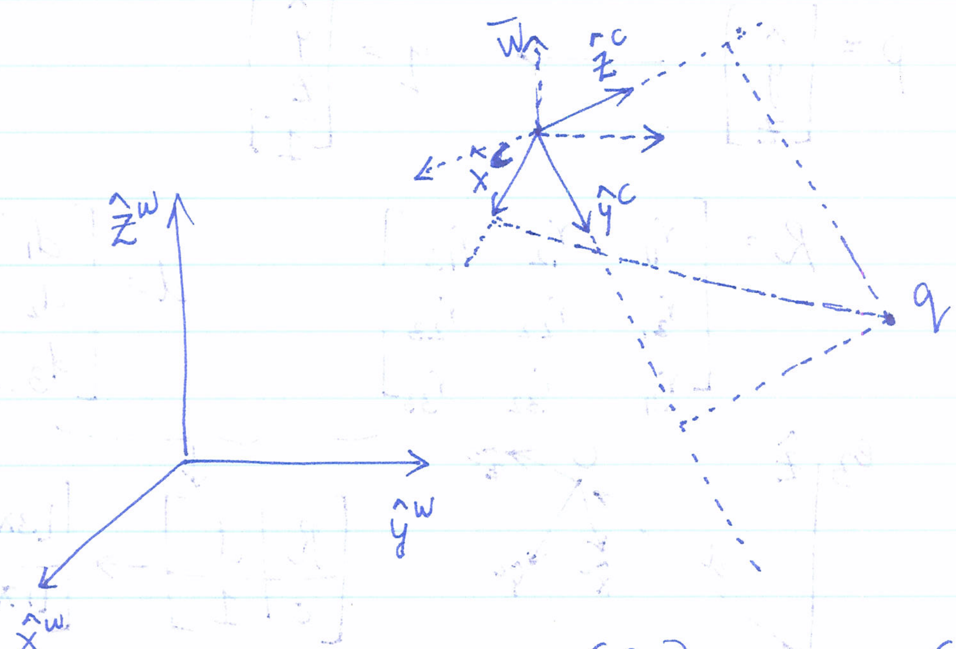
<u>properties:</u> R is invertible (orthonormal)

$$R^T R = R R^T = 1$$

inverse is transpose of matrix

$$\det(R) = +1$$

same orientation, physically realistic



$$q^c = \begin{Bmatrix} a \\ b \\ c \\ \hline 1 \end{Bmatrix} \qquad q^w = \begin{Bmatrix} 0 \\ x \\ 0 \\ \hline 1 \end{Bmatrix}$$

given point seen from camera q^c, what does the point "look like" from the world's perspective q^w?

$$Rp^c = R\begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = r_1 a \hat{x}^w + r_{12} b \hat{x}^w + r_{13} c \hat{x}^w$$

$$+ r_{21} a \hat{y}^w + r_{22} b \hat{y}^w + r_{23} c \hat{y}^w$$

$$+ r_{31} a \hat{z}^w + r_{32} b \hat{z}^w + r_{33} c \hat{z}^w$$

$$= (r_{11} a + r_{12} b + r_{13} c) \hat{x}^w$$

$$+ (r_{21} a + r_{22} b + r_{23} c) \hat{y}^w$$

$$+ (r_{31} a + r_{32} b + r_{33} c) \hat{z}^w$$

$$\Rightarrow p^{\bar{w}} = \begin{Bmatrix} r_{11} a + r_{12} b + r_{13} \cancel{b} c \\ r_{21} a + r_{22} b + r_{23} c \\ r_{31} a + r_{32} b + r_{33} c \end{Bmatrix}^{\bar{w}}$$

(when rewritten coordinates of q to be consistent with orientation of the world coordinate axes)

Next step is to compensate for translation:

$$p^w = R p^c + d$$

$$\Rightarrow \quad q^w = \begin{Bmatrix} R p^c + d \\ \hline 1 \end{Bmatrix}$$

— <u>Homogeneous</u>: $q^w = g . q^c$

$$q^w = \left[\begin{array}{c|c} R & d \\ \hline 0 & 1 \end{array}\right] \cdot \cancel{q^c} \begin{bmatrix} p_c \\ \hline 1 \end{bmatrix}$$

$$q^w = \begin{Bmatrix} R p^c + d \\ \hline 1 \end{Bmatrix}$$

- We did two transformations:

$$p^s = R^s p^w + d^s$$
$$= R^s R p^c + R^s d + d^s$$

$$q^s = g^s g \, q^c = \begin{bmatrix} R^s & d^s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix} q^c$$

$$= \begin{bmatrix} R^s R & R^s d + d^s \\ 0 & 1 \end{bmatrix} q^c$$

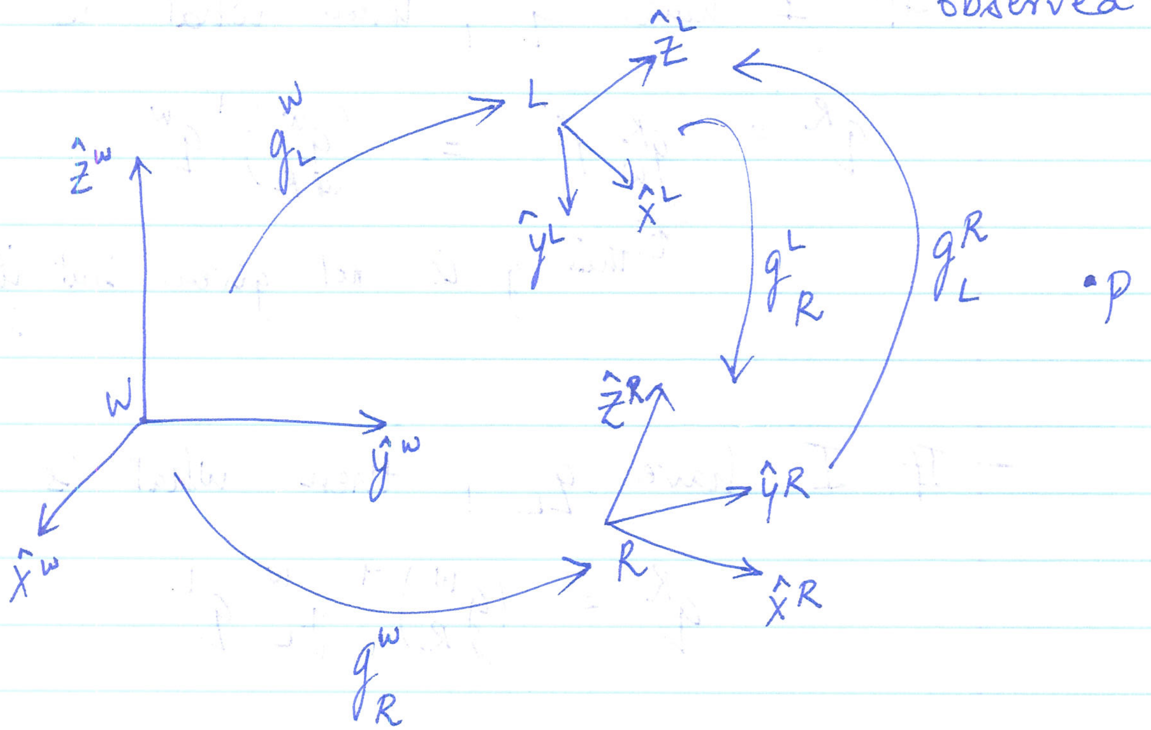
- What if I know what it was in the world coordinates and I needed it in the camera coordinates?

In homogeneous coordinates, just use matrix inverse

$$q^c = g^{-1} q^w$$

- There $\underline{\text{is a}}$ notation to help things out
(are several)

Superscript - coordinate frame/perspective of interest
Subscript - refers to the object/frame being observed



$$\left(g^L_R\right)^{-1} = g^R_L$$

given g^W_L and g^W_R, what is g^L_R?

$$g^L_R = \left(g^W_L\right)^{-1} g^W_R = g^L_W \, g^W_R = g^L_R$$

<u>others use</u>: ${}_W g_L$ for g^W_L

${}^W g_L$ for g^W_L

g_{WL} for g^W_L

- If I have q^W, then what is q^R?

$$q^R = \underline{g^R_W}\, q^W = \left(g^W_R\right)^{-1} q^W$$

↑ this g is not given but its inverse is given

- If I have q_L, then what is q^R?

$$q^R = \left(g^W_R\right)^{-1} g^W_L \, q^L$$

- Necessary to understand this geometry to get projection onto image correct.

- given q^w, where is it in images associated to left and right cameras?

Step 1) I need to know g_L^w & g_R^w

To get point w/ respect to camera frame

$$q^L = g_w^L q^w = (g_L^w)^{-1} q^w \quad \&$$

$$q^R = (g_R^w)^{-1} q^w$$

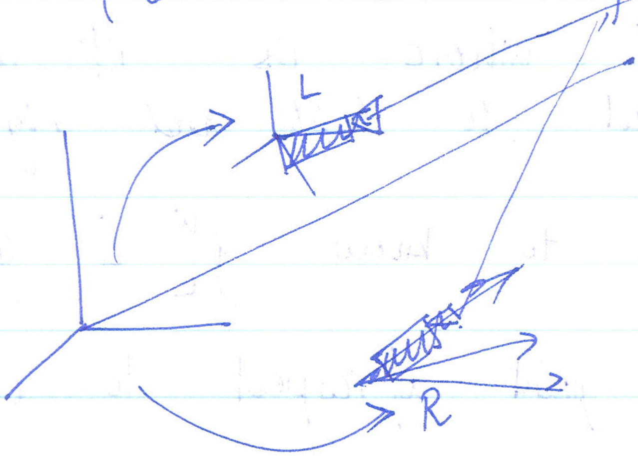
Step 2) Next step is projection:

$$\begin{Bmatrix} r_L^1 \\ r_L^2 \end{Bmatrix} = f_L \begin{Bmatrix} x^L/z^L \\ y^L/z^L \end{Bmatrix}$$

$$\begin{Bmatrix} r_R^1 \\ r_R^2 \end{Bmatrix} = f_R \begin{Bmatrix} x^R/z^R \\ y^R/z^R \end{Bmatrix}$$

Step 3) Pixelize it.

- given projections r_L and r_R, where is the point in the world?
  (inverse case: depth)



$$p^L = R^L_W p^W + d^L_W$$

$$p^R = R^R_W p^W + d^R_W$$

$$\Rightarrow \begin{Bmatrix} r^1_L \\ r^2_L \end{Bmatrix} = f_L \begin{Bmatrix} \left([R^L_W]_1 p^W + [d^L_W]_1\right) / \left([R^L_W]_3 p^W + [d^L_W]_3\right) \\ \left([R^L_W]_2 p^W + [d^L_W]_2\right) / \left([R^L_W]_3 p^W + [d^L_W]_3\right) \end{Bmatrix}$$

- same idea for right:

$$r^1_L\left([R^L_W]_3 p^W + [d^L_W]_3\right) - f_L\left([R^L_W]_1 p^W + [d^L_W]_1\right) = 0$$

$$r^2_L\left([R^L_W]_3 p^W + [d^L_W]_3\right) - f_L\left([R^L_W]_2 p^W + [d^L_W]_2\right) = 0$$

$$r_R^1\left([R_W^R]_3 p^W + [d_W^R]_3\right) - f_R\left([R_W^R]_1 p^W + [d_W^R]_1\right) = 0$$

$$r_R^2\left([R_W^R]_3 p^W + [d_W^R]_3\right) - f_R\left([R_W^R]_2 p^W + [d_W^R]_2\right) = 0$$

why keep all 4 equations?

1. Horizontal difference $\left(\frac{x+b/2}{z}, \frac{y}{z}\right)$

   $\left(\frac{x-b/2}{z}, \frac{y}{z}\right)$

2. Vertical difference $\left(\frac{x}{z}, \frac{y+b/2}{z}\right)$

   $\left(\frac{x}{z}, \frac{y-b/2}{z}\right)$

- need all 4 just in case two are redundant (then I loose rank).
  $\Rightarrow$ still have 3 unique equations and 3 unknowns.