ECE 4560 12/02/07

(Servo) ~~Error~~ Driven Control

Suppose that the desired trajectory were a constant reference, $\alpha^*(t) = \alpha^*$, then the control law becomes

$$\tau = M(\alpha)(-k_p e - k_v \dot{e}) + C(\alpha, \dot{\alpha})\dot{\alpha} + N(\alpha, \dot{\alpha})$$

Sometimes one doesn't want to deal with the complications of $M(\alpha)$ or doesn't want to deal with changing/shifting parameters in $M(\alpha)$, so it gets ignored,

$$\tau = -k_p e - k_v \dot{\alpha} + N(\alpha)$$

where $N(\alpha) = N(\alpha, 0)$

- Lyapunov analysis can be used to show that this a globally, asymptotically stable control law.
- simple & common control law
- Some manipulator manufacturers just stick with a law like this. Of course, performance may not be ~~optional~~ optimal.

What if the model is incorrect (which is pretty much a guarantee)?

– consider an error in the model, which will be described as unmodelled joint torques

$$\tau + \tau_d = M(\alpha)\ddot{\alpha} + C(\alpha, \dot{\alpha})\dot{\alpha} + N(\alpha, \dot{\alpha})$$

"disturbance" torque due to bad model.

$\Rightarrow$ using computed torque control

$$\ddot{e} + k_v \dot{e} + k_p e = M^{-1}(\alpha)\tau_d$$

converts group element to a velocity vector.

error in the Lie group.

position error: $K_p \ln \underbrace{g_e^{-1}(\alpha(t))\, g_e^*(t)}$

velocity error: $K_v\left(\xi_e^b(t) - (\xi_e^*)^b(t)\right)$

this is $(\xi_e^*)^b(t) = (g_e^*)^{-1}(t)\, \dot{g}_e^*(t)$

but the error feedback values are still in the Lie ~~group~~ algebra. They are not joint velocities.

Q: How do we get them into the joint space?
A: Using the Jacobian (inverse).

This leads to:

$$\tau = M(\alpha)\left[\ddot{\alpha}^*(t) - (J^b(\alpha))^{\#} K_p \ln\left(g_e^{-1}(\alpha)\, g_e^*(t)\right) - (J^b(\alpha))^{\#} K_v\left(\xi_e^b - (\xi_e^*(t))^b\right)\right]$$

be careful here. sign maybe wrong.

What about this part?

we know:

$$\dot{\alpha}^* = (J^b(\alpha))^{\#}(\xi_e^*(t))^b$$

$\Rightarrow$ differentiate:

$$\ddot{\alpha}^* = (J^b(\alpha))^{\#}(\dot{\xi}_e^*(t))^b + \left(\frac{d}{dt}(J^b(\alpha))^{\#}\right)(\xi_e^*(t))^b$$

$$\Rightarrow \tau = M(\alpha)\left[(J^b(\alpha))^{\#}(\dot{\xi}_e^*(t))^b - (J^b(\alpha))^{\#} K_p \ln\left(g_e^{-1}(\alpha)\, g_e^*(t)\right) - (J^b(\alpha))^{\#} K_v\left(\xi_e^b - (\xi_e^*(t))^b\right)\right] + M(\alpha)\left(\frac{d}{dt}(J^b(\alpha))^{\#}\right)(\xi_e^*(t))^b + C(\alpha,\dot{\alpha})\dot{\alpha} + N(\alpha,\dot{\alpha})$$

. recall that $\xi_E^b = J(\alpha)\dot{\alpha}$

ECE 4560

So, the first Cartesian-based controller is

$$\tau = M(\alpha)\left(J^b(\alpha)\right)^{\#}\left[\left(\dot{\xi}_e^*(t)\right)^b - K_p \ln\left(g_e^{-1}(\alpha)\, g_e^*(t)\right) - K_v\left(J^b(\alpha)\dot{\alpha} - (\xi_e^*(t))^b\right)\right]$$

$$+ \left[M(\alpha) D_\alpha\left(J^b(\alpha)\right)^{\#}\left(\xi_e^*(t)\right)^b + C(\alpha,\dot{\alpha})\right]\dot{\alpha} + N(\alpha,\dot{\alpha})$$

recall: $\left(\frac{d}{dt}\left(J^b(\alpha)\right)^{\#}\right)\left(\xi_e^*(t)\right)^b = D_\alpha\left(J^b(\alpha)\right)^{\#}\left(\xi_e^*(t)\right)^b \cdot \dot{\alpha}$

Can we prove that this works?
Yes, but it's a bit of trouble.

Start: $M(\alpha)\ddot{\alpha} + C(\alpha,\dot{\alpha})\dot{\alpha} + N(\alpha,\dot{\alpha}) = \tau$

Convert all terms to Lie group (g) and Lie algebra (ξ) terms.

$$M(\alpha)\left(\left(J^b(\alpha)\right)^{\#}\dot{\xi}^b + D_\alpha\left(J^b(\alpha)\right)^{\#}\cdot\xi^b\cdot\dot{\alpha}\right) + C(\alpha,\dot{\alpha})\dot{\alpha} + N(\alpha,\dot{\alpha}) = \tau$$

$\boxed{\begin{matrix}\tau = J^T(\alpha)F^b \\ \xi^b = J(\alpha)\dot{\alpha}\end{matrix}}$

use transpose of Jacobian properly.

$$\left(\left(J^b(\alpha)\right)^T\right)^{\#} M(\alpha)\left(J^b(\alpha)\right)^{\#}\dot{\xi}^b + \left(\left(J^b(\alpha)\right)^T\right)^{\#}\left[M(\alpha) D_\alpha\left(J^b(\alpha)\right)^{\#}\xi^b + C(\alpha,\dot{\alpha})\right]\dot{\alpha}$$

$$+ \left(\left(J^b(\alpha)\right)^T\right)^{\#} N(\alpha,\dot{\alpha}) = F^b$$

Define

$\tilde{M}(\alpha) = \left(\left(J^b(\alpha)\right)^T\right)^{\#} M(\alpha)\left(J^b(\alpha)\right)^{\#}$

$\tilde{N}(\alpha,\dot{\alpha}) = \left(\left(J^b(\alpha)\right)^T\right)^{\#} N(\alpha,\dot{\alpha})$

$$\Rightarrow \tilde{M}(\alpha)\dot{\xi}^b + \left(\left(J^b(\alpha)\right)^T\right)^{\#}\left[M(\alpha) D_\alpha\left(J^b(\alpha)\right)^{\#}\xi^b + C(\alpha,\dot{\alpha})\right]\dot{\alpha} + \tilde{N}(\alpha,\dot{\alpha}) = F^b$$

$\Rightarrow F^b = \left(\left(J^b(\alpha)\right)^T\right)^{\#}$ & move to left $\Rightarrow$

$$\tilde{M}(\alpha)\left[\dot{\xi}^b - (\dot{\xi}^*(t))^b + k_p \ln(g_e^{-1}(\alpha)\, g_e^*(t)) + k_v(\xi^b - (\xi^*(t))^b)\right]$$

$$+ ((J^b(\alpha))^T)^{\#}\left[\underbrace{M(\alpha)\, D_\alpha (J^b(\alpha))^{\#} (\xi^b - (\xi^*(t))^b)}_{\text{how do we deal with this?}}\right]\dot{\alpha} = 0$$

We can't so we'll be tricky.

$$\left[\dot{\xi}^b - (\dot{\xi}^*(t))^b + k_p \ln(g_e^{-1}(\alpha)\, g_e^*(t)) + k_v(\xi^b - (\xi^*(t))^b)\right]$$

$$+ (\tilde{M}(\alpha))^{-1}(J^b(\alpha))^T\left[M(\alpha)\, D_\alpha (J^b(\alpha))^{\#}(\xi^b - (\xi^*(t))^b)\right]\dot{\alpha} = 0$$

this cancels out parts here

$$\Rightarrow \left[\dot{\xi}^b - (\dot{\xi}^*(t))^b + k_p \ln(g_e^{-1}(\alpha)\, g_e^*(t)) + k_v(\xi^b - (\xi_e^*(t))^b)\right]$$

$$+ J^b(\alpha)\, D_\alpha (J^b(\alpha))^{\#}(\xi^b - (\xi^*(t))^b)\dot{\alpha} = 0$$

if I have $A(x)\cdot V$ then $(D_x A\cdot V)\cdot w = (D_x A\cdot w)\cdot V$

$$\Rightarrow \dot{\xi}^b - (\dot{\xi}^*(t))^b + \left[k_v + J^b(\alpha)\, D_\alpha (J^b(\alpha))^{\#}\dot{\alpha}\right](\xi^b - (\xi^*(t))^b) +$$

$$k_p \ln(g_e^{-1} g_d) = 0$$

$\Rightarrow$ looks like

$$\ddot{e}_g^b + \left[k_v + J^b(\alpha)\, D_\alpha (J^b(\alpha))^{\#}\dot{\alpha}\right]\dot{e}_g^b + k_p \ln e_g = 0$$

This is stable for the correct choice of k_v and k_p symmetric & positive definite.

Another good and simpler controller is:

$$\tau_c = M(\alpha)(J^b(\alpha))^{\#}\left[(\dot{\xi}^*(t))^b - k_v(\xi^b - (\xi^*(t))^b) - k_p \ln(g_e^{-1}(\alpha) g)\right]$$

$$+ C(\alpha,\dot{\alpha})\dot{\alpha} + N(\alpha,\dot{\alpha})$$