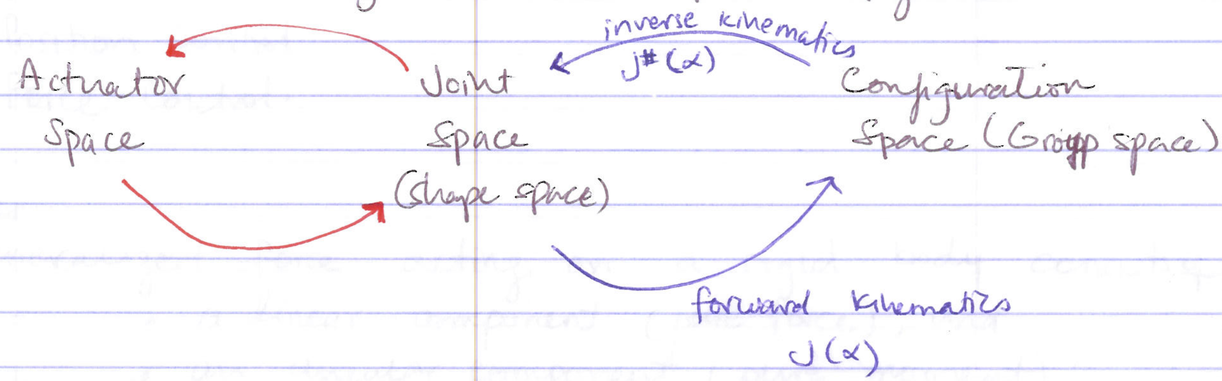
ECE 4560 11/26/07

Dynamics of Manipulators

• Kinematic models relate joint position/motion to end-effector position/motion.

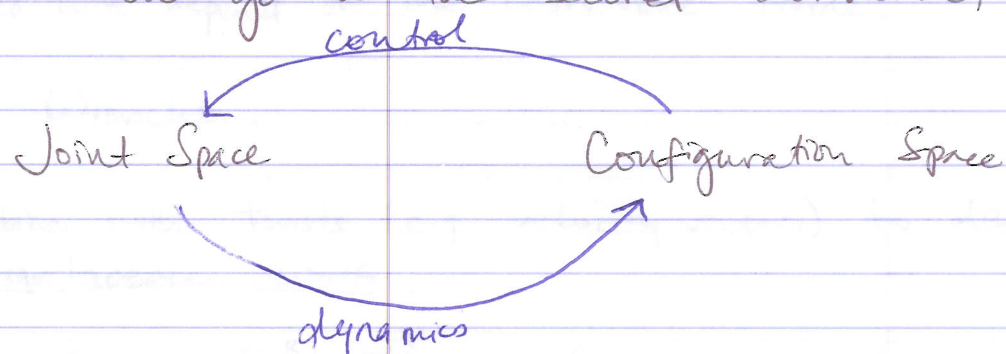
This was visualized earlier by the figure



• by dynamics of a manipulator, we mean how the manipulator moves in response to actuator forces/torques.

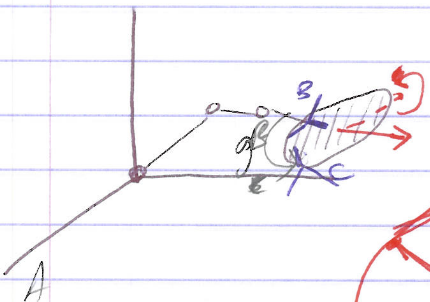
• at this point, we are ignoring the red arrows (left side in the figure), which really deals with control of servo-mechanisms. This is more of a classical control problem. Ignoring the mechanics of actuation allows ~~for~~ us to focus on the mechanics of manipulators.

. We want to get a better handle on what happens if we now go to the second derivative,



2) overtime SW generates work: $W = \int_{t_1}^{t_2} F_B^B \cdot \xi_B^B \, dt$

3) two wrenches are equivalent if they do the same work independent of ξ (the wrenches need not be applied at the same point).



F_B^A - wrench at point B in frame A

F_C^A — " " " C " " "

ξ_B^A — twist at point B in frame A

ξ_C^A — " " " C " " "

Same twist, but examined at different coordinate frame location.

$\xi_B^A = Ad_{g_C^B}\, \xi_C^A$

Equivalency means:

$$F_C^A \xi_C^A = F_B^A \cdot \xi_B^A$$

$$= \cancel{\ldots}$$

$$= F_B^A \, Ad_{g_C^B}\, \xi_C^A$$

$\Rightarrow$

$$F_C^A \xi_C^A = \left(Ad_{g_C^B}^T F_B^A \right) \cdot \xi_C^A$$

$a \cdot Bb$

$= a^T(Bb)$

$= (a^T(B^T)^T)b$

$= (B^T a)^T b$

$= (B^T a) \cdot b$

holds for any choice of ξ_C^A.

$\Rightarrow$

$$F_C^A = Ad_{g_C^B}^T F_B^A$$ ← transformation of wrench under change of frame & application point.

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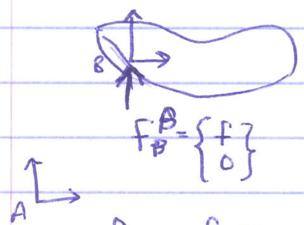
from last time,

$$\begin{Bmatrix} f_C^C \\ \tau_C^C \end{Bmatrix} = \begin{bmatrix} R_{BC}^T & 0 \\ -R_{BC}^T \hat{d}_{BC} & R_{BC}^T \end{bmatrix} \begin{Bmatrix} f_B^B \\ \tau_B^B \end{Bmatrix} \qquad \text{for } g_C^B = (R_{BC}, d_{BC})$$

Example



$F_B^B = \begin{Bmatrix} f \\ 0 \end{Bmatrix}$

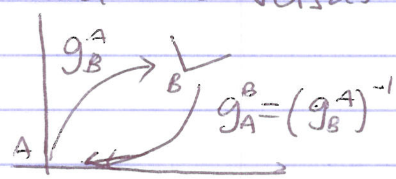
$g_C^B = (\mathbb{1}, d_{BC})$

$\Rightarrow F_C^C = \begin{bmatrix} \mathbb{1} & 0 \\ -\hat{d}_{BC} & \mathbb{1} \end{bmatrix} \begin{Bmatrix} f \\ 0 \end{Bmatrix}$

Pure force at B $\longrightarrow$ induces a torque at C $= \begin{Bmatrix} f \\ -\hat{d}_{BC} f \end{Bmatrix} = \begin{Bmatrix} f \\ -d_{BC} \times f \end{Bmatrix}$

Can go from body to spatial (and vice-versa)

$$F_A^A = Ad^T_{g_A^B} F_B^B$$



if A is spatial location (inertial coordinate origin) and B is body location, then g_A^B is transformation from body to origin

$\Rightarrow F^s = Ad^T_{g^{-1}} F^b$

$= (Ad_g^T)^{-1} F^b$

$\Rightarrow F^b = Ad_g^T F^s$

this is flipped when compared to what we are used to, which is $\xi^s = Ad_g \xi^b \Rightarrow \xi^b = Ad_{g^{-1}} \xi^s$

- Wrenches can be added, but only if they are applied at the same point.

Theorem (Poinsot). Every collection of wrenches applied to a rigid body is equivalent to a force applied along a fixed axis, plus a torque about the same axis.

- this is the dual theorem to Chasles' Theorem regarding twists.

Control of Manipulators

from linear control theory:

$$\dot{x} = Ax + Bu \qquad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m$$

- system is ~~controllable~~ controllable when

$$[B \;\; AB \;\; A^2B \ldots A^{n-1}B] \text{ has full rank.}$$

- if system is controllable, then the poles of the system can be arbitrarily placed using statespace feedback $u = -kx$ where K is pos. def.

So, stabilizing feedback ~~~~ of the form $u = -Kx$ takes system to the origin.

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from last time,

$\dot{x} = Ax + bu \quad , \quad x \in \mathbb{R}^n \quad u \in \mathbb{R}^m$

- if controllable, then ~~feedb~~ feedback control law, $u = -Kx$ for some K can stabilize system to origin.

- for stabilization to set point or to trajectory, need to include feedforward input
  $\Rightarrow$ feedforward + feedback

send it in right direction
guarantee it gets there (given unknown)

For our system, we have the equations of motion being of the form

$$M(\alpha)\ddot{\alpha} + C(\alpha, \dot{\alpha})\dot{\alpha} + N(\alpha, \dot{\alpha}) = \tau$$

$M(\alpha)$: $m \times m$ symmetric matrix depending on α
$C(\alpha, \dot{\alpha})$: $m \times m$ skew-symmetric matrix depending on α and linearly on $\dot{\alpha}$
$N(\alpha, \dot{\alpha})$: $m \times 1$ vector
τ: $m \times 1$ vector of torques.

- derived from a Langrangian, $L = KE - PE$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\alpha}} - \frac{\partial L}{\partial \alpha} = \tau$$

- this is a nonlinear system, so linear control won't be ~~[illegible]~~ so effective.
- given that, is there some way to get it to look linear?

Feedforward Control

Suppose we had equations of motion & a desired joint trajectory to track, $\alpha^*(t)$

If $\alpha(0) = \alpha^*(0)$ and $\dot{\alpha}(0) = \dot{\alpha}^*(0)$, then choosing

$$\tau = M(\alpha^*)\ddot{\alpha}^* + C(\alpha^*, \dot{\alpha}^*)\dot{\alpha}^* + N(\alpha^*, \dot{\alpha}^*)$$

would cause $\alpha(t)$ to follow the same trajectory as $\alpha^*(t)$

α and α^* have same differential equation $\Rightarrow$ same trajectory if initial conditions match.

But, the technique fails if

1) initial conditions do not match, $\alpha(0) \neq \alpha^*(0)$ or $\dot{\alpha}(0) \neq \dot{\alpha}^*(0)$.
2) our model is incorrect (wrong masses, lengths, or model lacks friction, or ...)
3) the system experiences external disturbances.

Can we avoid this problem? YES! THROUGH FEEDBACK

Computed Torque Control:

Consider a control torque that
- cancels nonlinearities
- applies desired trajectory torque

given measurements of α and $\dot{\alpha}$.

$$\tau = M(\alpha)\ddot{\alpha}^* + C(\alpha, \dot{\alpha})\dot{\alpha} + N(\alpha, \dot{\alpha})$$

Now we are using state values as measured using sensors.

$\Rightarrow$ plugging into equations of motion gives,

$$M(\alpha)\ddot{\alpha} = M(\alpha)\ddot{\alpha}^*$$
$$\Rightarrow \ddot{\alpha} = \ddot{\alpha}^*$$

Will result in the desired acceleration in the face of state uncertainty. Slightly more robust, but still not good enough. We need error feedback.

Notice that the system is now linear

$$\ddot{\alpha} = \ddot{\alpha}^*$$

What if we augment this controller with an error feedback law?

$$\tau = M(\alpha)(\ddot{\alpha}^* - k_p e - k_v \dot{e}) + C(\alpha,\dot{\alpha})\dot{\alpha} + N(\alpha,\dot{\alpha})$$

where,

$e(t) = \alpha(t) - \alpha^*(t)$, $\dot{e}(t) = \dot{\alpha}(t) - \dot{\alpha}^*(t)$, and k_p & k_v are constant gain matrices.

$\Rightarrow$ plug into equations of motion

$$M(\alpha)(\ddot{e} + k_v \dot{e} + k_p e) = 0$$

$$\Rightarrow M(\alpha) > 0$$
$$\ddot{e} + k_v \dot{e} + k_p e = 0$$

i.e. this is a linear ordinary diff. equation, stable for appropriate choices of k_p & k_v.

The control law can be broken up into two contributions:

$$\tau = \underbrace{M(\alpha)\ddot{\alpha}^* + C(\alpha,\dot{\alpha})\dot{\alpha} + N(\alpha,\dot{\alpha})}_{\text{feed forward, } \tau_{FF}} + \underbrace{M(\alpha)(-k_p e - k_v \dot{e})}_{\text{feedback, } \tau_{FB}}$$

- very typical approach for control of nonlinear systems.
- choose gains k_p & k_v to achieve a given response (from linear systems theory).