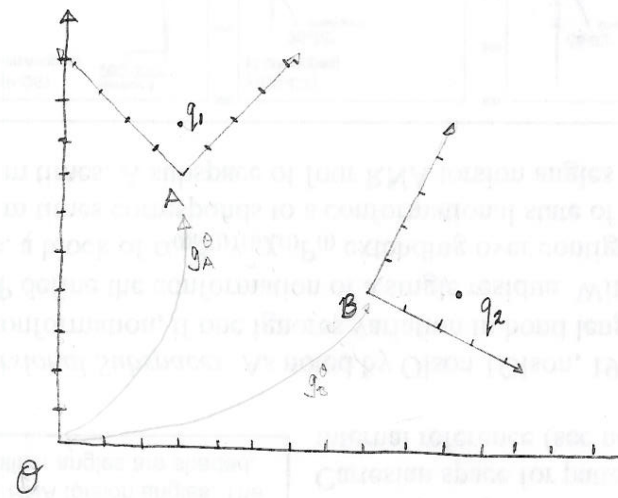
COORDINATES - FRAMES AND POINTS

A coordinate frame is specified by the location of its origin and the rotation of its axes (relative to some other frame).



O - global frame.

A - another reference frame.

B - yet another reference frame.

Any time a point coordinate is given, the frame associated to its coordinates must be given. The frames depicted are O, A & B.

$$g_A^O = (3, 7, \pi/4)$$

$\pi/4$ → rotation by $\pi/4$ (45°)

(3, 7) → translation by (3, 7)

} coordinate frame A relative to frame O

$$g_B^O = (8, 4, -\pi/6)$$

$-\pi/6$ → rotation by $-\pi/6$ (-30°)

(8, 4) → translation by (8, 4)

} coordinate frame B relative to frame O

What is g_θ^θ? It is location of frame θ relative to frame θ.

Well, it is referring to itself!

To get to frame θ from frame θ requires doing nothing

$\Rightarrow$ no rotation, no translation

$\Rightarrow$

$$g_\theta^\theta = (0, 0, 0)$$

We can also ask for the location of frame B relative to frame A.

This is

$$g_B^A = (\underbrace{\sqrt{2}, -4\sqrt{2}}_{\text{translation by } (\sqrt{2},\, -4\sqrt{2})}, \underbrace{-5\pi/12}_{\text{Rotate by } -5\pi/12 \ (-75^\circ)})$$

CAN YOU SEE WHY?

Now, what about the point g_1? Where is it?

Well, from the figure, we see $g_1^A = \{1\ 1\}^A$

That means

$$g_1^\theta = g_A^\theta * g_1^A = \left(d_{\theta A}^\theta, R_A^\theta(\theta_{\theta A})\right) * g_1^A$$

$$= d_{\theta A}^\theta + R_A^\theta(\theta_{\theta A}) \cdot g_1^A$$

$$= \begin{Bmatrix} 3 \\ 7 \end{Bmatrix}^\theta + \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}_A^\theta \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}^A = \underbrace{\begin{Bmatrix} 3 \\ 7+\sqrt{2} \end{Bmatrix}^\theta}_{\text{location of } g_1 \text{ in } \theta\text{'s frame.}}$$

Where is q_1 with respect to frame B?

To get that we need to solve for:

$$q_1^B = g_A^B * q_1^A$$

but we only know g_A^0 & g_B^A.

Fortunately, we also know that $g_A^B = (g_B^A)^{-1}$

$$q_1^B = (g_B^A)^{-1} q_1^A$$

NOTE:

$$R_B^A = R\left(-\tfrac{5\pi}{12}\right) = \begin{bmatrix} \cos(5\pi/12) & \sin(5\pi/12) \\ -\sin(5\pi/12) & \cos(5\pi/12) \end{bmatrix}$$

$$= \left(-(R_B^A)^{-1} d_{AB}^A ,\ (R_B^A)^{-1} \right) * q_1^A$$

$$= -\begin{bmatrix} \cos(5\pi/12) & -\sin(5\pi/12) \\ \sin(5\pi/12) & \cos(5\pi/12) \end{bmatrix}_A^B \cdot \begin{Bmatrix} \sqrt{2} \\ -4\sqrt{2} \end{Bmatrix}^A + \begin{bmatrix} \cos(5\pi/12) & -\sin(5\pi/12) \\ \sin(5\pi/12) & \cos(5\pi/12) \end{bmatrix}_A^B \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}^A$$

$$= \begin{bmatrix} \cos(5\pi/12) & \sin(5\pi/12) \\ -\sin(5\pi/12) & \cos(5\pi/12) \end{bmatrix}_A^B \begin{Bmatrix} 1-\sqrt{2} \\ 1+4\sqrt{2} \end{Bmatrix}^A$$

$$= \begin{Bmatrix} -6.5372 \\ 1.3228 \end{Bmatrix}$$

How else could you have found q_1^B? ... HINT

Same kinds of questions can be asked about q_2.

How would you answer given $q_2^B = \begin{Bmatrix} 2 \\ 1 \end{Bmatrix}^B$?

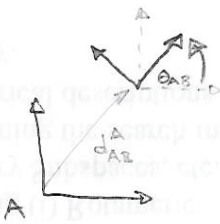
(e.g. what are q_2^A and q_2^0?)

A: Using g_0^B & g_0^A, or more directly $(g_0^B)^{-1} q_1^0$.

SYNOPSIS:

a coordinate frame (B) is given by a translation and a rotation relative to some other frame (A),

$$g_B^A = (d_{AB}^A, R_B^A)$$



$R_B^A = R(\theta_{AB})$

$$R(\theta_{AB}) = \begin{bmatrix} \cos(\theta_{AB}) & -\sin(\theta_{AB}) \\ \sin(\theta_{AB}) & \cos(\theta_{AB}) \end{bmatrix}$$

to get $\overbrace{\text{frame (A) with respect to frame (B)}}^{\text{this is } g_A^B}$ given g_B^A, just use the inverse,

$$g_A^B = (g_B^A)^{-1} = (d_{AB}^A, R_B^A)^{-1}$$

$$= (-(R_B^A)^{-1} d_{AB}^A, (R_B^A)^{-1})$$

$$\left(\text{NOTE: } (R_B^A)^{-1} = R^{-1}(\theta_{AB}) = R(-\theta_{AB})\right)$$

JUST A QUICK LITTLE CONVENIENCE FOR THE PLANAR CASE.

The identity displacement is $e = (0, 1)$

in vector notation, this is $e = (0, 0, 0)$.

So, in vector/matrix form, we have:

$$g = (\vec{d}, R)$$

identity $\quad e = (0, \mathbb{1})$

$\quad$ ↑ zero vector $\quad$ ↑ identity matrix.

multiplication $\quad g_1 \cdot g_2 = (\vec{d}_1, R_1) \cdot (\vec{d}_2, R_2) = (\vec{d}_1 + R_1\vec{d}_2, R_1 R_2)$

inverse $\quad g^{-1} = (\vec{d}, R)^{-1} = (-R^{-1}\vec{d}, R^{-1})$

to transform points: $\quad g \cdot p = (\vec{d}, R) \cdot p = \vec{d} + Rp$.

In vector form, we have:

$$g = (x, y, \theta) \text{ or } (\vec{d}, \theta) \text{ where } \vec{d} = \begin{Bmatrix} x \\ y \end{Bmatrix}.$$

identity $\quad e = (0,0,0) \text{ OR } (\vec{0}, 0)$

multiplication $\quad g_1 \cdot g_2 = (\vec{d}_1, \theta_1) \cdot (\vec{d}_2, \theta_2) = (\vec{d}_1 + R(\theta_1)\vec{d}_2, \theta_1 + \theta_2)$

where $R(\theta_1) = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) \end{bmatrix}$

inverse $\quad g^{-1} = (\vec{d}, \theta)^{-1} = (-R(-\theta)\vec{d}, -\theta)$

to transform points $\quad g \cdot p = (\vec{d}, \theta) \cdot p = \vec{d} + R(\theta) \cdot p$

to go from vector form to vector/matrix form

$$(\vec{d}, \theta) \mapsto (\vec{d}, R(\theta))$$

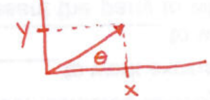
to go from vector/matrix form to vector form

$$(\vec{d}, R) \mapsto (\vec{d}, \operatorname{atan2}(R_{21}, R_{11}))$$

where $R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}$

· this is the Matlab version of the ~~inverse~~ functional inverse to the tangent.

$\theta = \operatorname{atan2}(y, x)$ for the picture below:



In our case, this leads to

$\theta = \operatorname{atan2}(R_{21}, R_{11})$